

# Cable Force Analysis for Bridge Completion and Construction of Railway Long-Span Continuous Beam and Arch Bridge

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**Abstract:** The continuous beam-arch composite bridge is a typical ternary structure. Due to its internal high-order static indeterminacy, the three parts of the structure are restricted during the construction stage and the later stage of the bridge, and the tensile force of the suspender has a great impact on the final stress state of itself and the overall structure. This article relies on engineering examples, using the Midas Civil finite element software and the minimum bending strain energy method to calculate the bridge cable force, and analyzes the static characteristics of the structure under this cable force. The difference method is used to calculate the construction cable force; changes of cable force as well as the stress and deformation of the beam and arch structure during the tension process are analyzed. The results show that, the cable force obtained by the minimum bending energy method is relatively uniform. Under this cable force, the arch rib steel tube is under uniform compression, and the stress of the main beam is deformed and meets the requirements of the code. Based on the minimum bending energy method, the cable forces of bridge are calculated for the proposed tensioning sequence. After the second phase of paving, the cable force reached the bridge cable force, and the accuracy met the requirements. The influence of the boom force on the static characteristics of the beam and arch during the tensioning process was analyzed.

## 1. Introduction

Continuous beam-arch bridge has a wide range of applications, good and stable economic indicators, and can give full play to the advantages of concrete and arch. It has more obvious competitiveness than other bridges with the same span.[1] Zhiwei Sun [2] proposed a method to determine the reasonable suspender force based on the stress balance method, which can make the bending moment of the arch rib section locates within the feasible region of the bending moment of the arch rib section. It which can determine the reasonable suspender force, and make the stress state of the whole arch beam composite structure in the optimal state. Kunquan Ma [3] made a detailed stress analysis on the joint part of arch and beam of high-speed railway continuous beam flexible arch bridge, and put forward design and construction suggestions.

Continuous beam-arch composite bridge has common advantages of continuous beam and concrete-filled steel tube due to its strong crossing capacity and large structural stiffness. In order to achieve the purpose of more coordinated deformation and more balanced stress of the beam and arch structure in the construction process and the bridge completion stage, the suspender is generally tensioned twice or in batches. When a group of suspenders is tensioned, it will produce certain impacts on suspenders which has been tensioned. The multiple tensioning will increase the construction cost and prolong the construction period, and the impact on the structure will become more complex.[4] Therefore, under the condition of meeting the requirements of the specification, in the construction process, the times of tensioning the suspender should be reduced as far as possible. It is of great significance to the requirements of construction schedule and the construction cost control. [5]

The reasonable inner distribution and its size under the completed state is one of the criteria to measure the continuous beam-arch bridge. Zhongping Liu[6] studied the influence of three design parameters, namely the rise span ratio, the shape of arch axis and the arch beam stiffness ratio, on the mechanical characteristics of railway continuous beam-arch composite bridge. Based on the

analysis method for cable-stayed bridges, this paper analyzes the railway continuous beam arch composite bridge. The cable force of the completed bridge and the construction cable force are analyzed by using the minimum bending energy method and the difference method respectively. The paper also analyzes the influence of the cable force on the static characteristics of the beam and arch structure after the completion of the bridge and in the process of suspender tension.

## 2. Overview of the Project

The main bridge of the Guangzhou-Shenzhen Hengyonghai Super Bridge is a (90 + 180 + 90) m continuous beam-arch composite bridge with the length of 361.85m. The main beam is a pre-stressed concrete structure with single box double chamber variable height box section. The calculated span of the arch rib is 180.0m; the design rise height  $f$  is 36.0m; the rise span ratio  $f / L$  is 1:5. The arch rib is a concrete-filled steel tube structure with dumbbell shaped section of the equal height. The section height is 3.1m and the chord diameter of  $\Phi$  is 1.1m. The transverse center distance between the two arch ribs is 13.1m. The spacing of suspenders along the bridge direction is 9m. There are 18 groups of double suspenders in the whole bridge. The suspender adopts parallel steel wire bundle and cold casting heading anchor. The upper end of the suspender passes through the arch rib and is anchored to the tension base at the upper edge of the arch rib, and the lower end is anchored to the lower edge of the lifting point beam to fix the base. The overall layout of the whole bridge is shown in Figure 1.

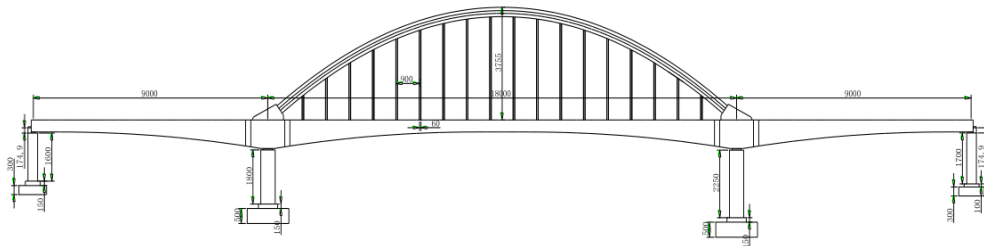


Fig.1 Overall Layout of the Bridge. (Unit: Cm)

The main construction sequence of the bridge is as follows. First, the main beam is poured with the hanging basket cantilever, and the corresponding pre-stress is tensioned. Second, the main beam is closed from the side to the middle, and all the pre-stressing force is tensioned. Third, the girder transport vehicle is used for 3 months. Fourth, the steel pipe arch rib is assembled with low support with the bridge deck as the working face. (The total load concentration of deck support and arch rib steel pipe is 50.2N/m, and the concentrated force of machines and equipment is 800kN in the middle of the span, which is similar to the actual construction situation.) The fifth step is using the bridge deck tower and other equipment to make the steel pipe arch rib vertical swivel in place, closing the arch crown and consolidating the arch foot. The sixth step is removing the temporary support of the bridge deck, removing the tower frame, the wind cable and other equipment for vertical rotation. Seventh, pouring concrete in the upper chord, the lower chord and the batten plate of the arch rib in turn. Eighth, installing the suspender, and adding the initial tension to the suspender according to the specified order,  $N=200\text{kN}$ ; adjusting the suspender cable force to  $N=200\text{kN}$ . Ninth, keeping the beam for 3 months and construct the bridge deck system. Tenth, adjusting the suspender force to the designed cable force of the completed bridge.

## 3. Establishment of the Finite Element Model

According to actual construction steps, corresponding construction stages are defined in Midas / Civil finite element analysis software. Except for the No.0 block, each construction block of the main beam is divided into a unit. There are 3708 nodes in the whole bridge model, and 9048 units are divided. Table 1 shows the type and quantity of elements used in each component of the model and the number of units divided by the component. In the model, block No. 0 and the permanent support of side pier is simulated by general support and elastic connection (rigid). The connection

between suspender and beam is simulated by rigid connection. Beam end release is simulated between the steel tube bracket and arch rib and between the steel tube bracket and the main beam. The connection between arches and the beam is simulated by general support and elastic (rigid) connection. The full bridge simulation model is shown in Figure 2.

Table 1 Materials and Units Used in Each Main Component.

No.	Type	Material	Elasticity modulus (MPa)	Unit class	Number of division units
1	Main beam	C55 concrete	3.55e4	Beam unit	121
2	Arch rib	Q345qD	2.06e5	Beam unit	344
3	Suspender	High strength steel wire	2.00e5	Truss unit	18
4	Transverse bracing	Q345qD	2.06e5	Beam unit	614
5	Arch rib bracket	Q235B	2.06e5	Beam unit	7062



Fig.2 Finite Element Model of the Hengyonghai Bridge.

#### 4. Common Methods of Reasonable Bridge Cable Force

##### 4.1 Rigid Support Continuous Beam Method

The rigid support continuous beam method is one of the earliest methods used by engineers to determine the reasonable cable force of a bridge. Its mechanical idea is clear, that is, the arch bridge under constant stress is regarded as a finite number of rigid support continuous beams at the bottom, and the stress states of the two are considered to be the same.[7] Similar to cable-stayed bridges and tied arch bridges, under the action of dead load and suspender force, the vertical displacement of the continuous beam - arch composite bridges will occur at the rigid support, that is, the anchorage point of the suspender and the beam body. In this method, the stress state of the arch bridge is regarded as the same as that of the continuous beam with rigid support at the bottom. In this way, the reaction force of the rigid support at the bottom of the continuous beam can be regarded as the tension of the suspender. The vertical displacement at the anchorage point between the suspender and the beam body is zero [8]. As shown in Figure 3, when solving the problem, all suspenders in the arch bridge are cut off, and the anchorage positions between suspenders and the beam body are replaced by the rigid support. The constraint at the original suspender is replaced. The rigid support reaction force under the dead load of the structure is solved, and the suspender force at the corresponding position is determined by the support reaction force.

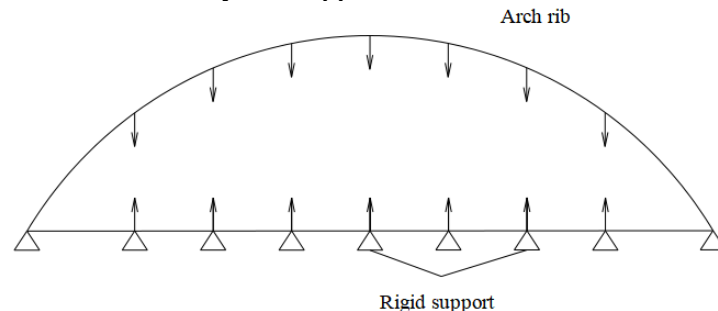


Fig.3 Schematic Diagram of Rigid Support Continuous Beam Method.

As shown in Figure 3, the suspender force (boom force) is replaced by the supporting reaction force of rigid support, and then the suspender force is calculated by structural mechanical force method after it is changed into a fundamental structure. Assuming that the reaction forces of rigid

supports at the first, second, third,..., and  $i$ th positions are  $F_1, F_2, F_3, \dots, F_i$ , after the structure is subjected to the dead load, and the redundant constraints are removed, the displacements at the anchorage points of suspenders and the beam body, namely the rigid supports, are  $\Delta_{1h}, \Delta_{2h}, \Delta_{3h}, \dots, \Delta_{ih}$ . Here, the left subscript is the location of the displacement, and the right subscript is the origin of the displacement. It is assumed that the displacement at the anchorage point under the action of unit force “1” is  $\delta_{ij}$ , which represents the displacement of unit force 1 in the basic structure at the point  $i$  under the single action of point  $j$ , the force method equation can be listed as follows.

$$\begin{aligned}\delta_{11}F_1 + \delta_{12}F_2 + \delta_{13}F_3 + \dots + \delta_{1i}F_i + \Delta_{1h} &= 0 \\ \delta_{21}F_1 + \delta_{22}F_2 + \delta_{23}F_3 + \dots + \delta_{2i}F_i + \Delta_{2h} &= 0 \\ \delta_{31}F_1 + \delta_{32}F_2 + \delta_{33}F_3 + \dots + \delta_{3i}F_i + \Delta_{3h} &= 0 \\ &\dots \\ \delta_{i1}F_1 + \delta_{i2}F_2 + \delta_{i3}F_3 + \dots + \delta_{ii}F_i + \Delta_{ih} &= 0\end{aligned}\quad (1)$$

The above is written in the matrix form as:

$$[\delta]\{F\} + [D] = 0 \quad (2)$$

In which:

$$[\delta] = \begin{bmatrix} \delta_{11} & \delta_{12} & \dots & \delta_{1i} \\ \delta_{21} & \delta_{22} & \dots & \delta_{2i} \\ \dots & \dots & \dots & \dots \\ \delta_{i1} & \delta_{i2} & \dots & \delta_{ii} \end{bmatrix} \quad (3)$$

The influence matrix  $[\delta]$  of the suspender and beam anchorage under the action of unit force “1” and the displacement  $[\Delta]$  of the suspender and beam anchorage under the action of constant load can be obtained by using the finite element analysis software. According to formula (2), the support reaction force at the rigid support can be calculated, and the cable force of the bridge under the action of constant load can be obtained. However, to calculate the cable force by this method, the beam bending moment is relatively small after the completion of the bridge. But the uneven cable force after the completion of the bridge makes the change of cable force prominent, and the overall distribution of suspender force is not ideal, which will add a lot of unnecessary troubles to the later construction.

## 4.2 Zero Displacement Method

This method means that under the joint action of dead load and cable force, the displacement of the connection point between suspender and collar beam is zero in the completed bridge stage. The cable force of the suspender of the completed bridge can be obtained under this condition, that is, the deformation coordination condition of this method is zero displacement of the connection point between the suspender and the collar beam. For the construction bridge with one-off frame, its stress system is similar to that of the rigid support continuous beam method to solve the cable force of the completed bridge, so the calculation results are also relatively close. The zero displacement method is not applicable to cantilever cast-in-place bridges.

## 4.3 Minimum Bending Energy Method

The purpose of this method is to reach a set of cable force values for tensioning the suspender and to minimize the objective function of the total bending strain energy of the beam and arch rib, so as to determine the reasonable cable force of the structure under constant load.[8][9] In this method, the process of calculating the internal force of the structure under constant load is also cutting the suspender and replace it with the corresponding force. The suspender forces of the first, second, third,...,  $i$ th suspender are  $T_1, T_2, T_3, \dots, T_i$ . When the value of suspender force  $T_i$  is 1, the

bending moment, the axial force and the shear force of each section of the arch bridge structure are  $\overline{M}_i$ ,  $\overline{F}_i$  and  $\overline{Q}_i$  respectively. The internal forces in each section are:

$$M = \sum_{i=1}^n T_i \overline{M}_i + M_0, \quad F = \sum_{i=1}^n T_i \overline{F}_i + F_0, \quad Q = \sum_{i=1}^n T_i \overline{Q}_i + Q_0 \quad (4)$$

Where,  $M_0$ ,  $F_0$  and  $Q_0$  are the bending moment, the shear force and the axial force generated by each section of the structure under constant load.

According to the mechanics of materials, when small deformation of the strut structure occurs within the elastic range, that is, the strain energy of the beam body and the arch rib is:

$$U = \int_0^l \frac{M^2(x)}{2EI} dx + \int_0^l \frac{F^2(x)}{2EA} dx + \int_0^l \frac{Q^2(x)}{2GI_p} dx \quad (5)$$

Since the objective function of this method is the bending strain energy, the second and third terms in formula (5) should be removed, so the strain energy only caused by the structural bending moment is,

$$U = \int_0^l \frac{M^2(x)}{2EI} dx \quad (6)$$

By substituting formula (4) into formula (6), it can be obtained that:

$$U = \int_0^l \frac{(\sum_{i=1}^n T_i \overline{M}_i + M_0)^2}{2EI} dx \quad (7)$$

In order to minimize the strain energy  $U$  in formula (7),  $U$  should be set as the standing value, namely

$$\frac{\partial U}{\partial T_i} = 0 \quad (8)$$

To solve formula (8), we can obtain the reasonable cable force.

## 5. Reasonable Cable Force Analysis

### 5.1 Cable Force Analysis of the Continuous Beam-Arch Composite Bridge

The rigid supported continuous beam method has a great influence on the stiffness of arch ribs. Uneven cable forces will be generated in solution. The zero displacement method is not applicable to cantilever cast main beams [10], so the minimum bending energy method is adopted in this section to calculate the cable force of the completed bridge based on the Midas Civil finite element model. The element parameters and construction stages have been clearly stated above. When calculating the cable force of the bridge by using the minimum bending energy method, the initial tension of the suspender is given arbitrarily, and then the section areas of the main beam, the arch rib and the suspender are enlarged by 10000 times. The cable force calculated is the cable force of the bridge, as shown in Figure 4. The deformation and stress of the main beam are obtained by bringing in the cable force of the completed bridge, as shown in Figure 5 and Figure 6.

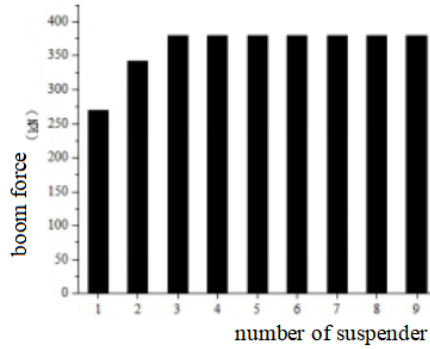


Fig.4 Le Force Diagram.

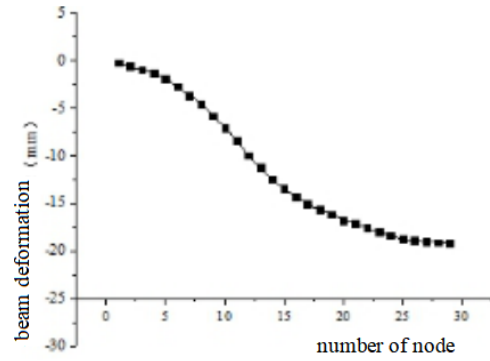


Fig 5 Deformation Diagram of Main Beam Mid-Span.



Fig.6 Stress Diagram of Main Beam and Arch Rib.

It can be seen from Figure 4 to Figure 6 that cable forces calculated by the minimum bending energy method are about 373kN except for the short suspender, which are relatively uniform. The arch rib steel pipe is under uniform compression, and the stress of main beam is from negative 1.5MPa to 14.66 MPa, which is within the allowable stress range of C55 concrete. The deformation of the main beam is smooth, and the maximum displacement occurs in the middle of the middle span. The value is -19.6mm, which meets the requirements of the specification.

## 5.2 Determination of Construction Cable Force of Continuous Beam-Arch Composite Bridge

Although it is clear and easy to solve the construction cable force by using the reverse demolition method or the reverse installation method, the methods do not consider problems of nonlinear structure and concrete shrinkage and creep. Although non-linear factors of the structure are considered in the reverse installation method, there are still some factors that are not in line with the actual situation after iteration. For example, the force of the same size and opposite direction is applied at the support position when the support is demolished, which is obviously inconsistent with the force at the front support without support in the actual construction.

The positive iteration method, also known as the difference method, is the linear iteration with the cable force as the objective function after considering the shrinkage and creep. The cable force of the completed bridge is input as the initial value. After several iterations, the obtained cable force value approximates to the bridge cable force until convergence, and the error of the cable force value is controlled at about 1.5%. The cable force obtained in each construction stage is the construction cable force considering concrete shrinkage and creep.[11] [12]

After eight iterations, we achieved a good convergence result, with the maximum error value of about 0.66%, which meets the requirement of error within 1.5%.The cable forces in each construction stage are shown in Table 2.

Table 2 Cable Force Value at Each Construction Stage. (1)

Position of suspender			Tensioning sequence		
	1	2	3	4	5
D1					
D2					
D3			222.1	(221.1)	(161.6)
D4					130.1
D5	344.1	(294.3)	(204.1)	(167.4)	(120.9)
D6					
D7				100.1	(91.6)
D8					
D9		85.7	(102.9)	(54.7)	(60.3)
D10		84.6	(101.7)	(54.3)	(59.8)
D11					
D12				101.6	(88.8)
D13					
D14	343.2	(293.6)	(201.8)	(166.1)	(119.7)
D15					130.8
D16			224.9	(223.6)	(163.3)
D17					
D18					

Table 2 Cable Force Value at Each Construction Stage. (2)

Position of suspender		Tensioning sequence			
	6	7	8	9	the second phase
D1				119.2	275.0
D2		127.2	(129.1)	(114.0)	342.0
D3	(165.1)	(123.3)	(119.9)	(118.8)	373.4
D4	(130.9)	(114.1)	(102.2)	(108.1)	373.2
D5	(117.9)	(114.2)	(91.7)	(100.2)	375.5
D6			74.1	(82.9)	373.1
D7	(73.6)	(76.7)	(57.3)	(65.8)	373.3
D8	49.3	(52.1)	(35.6)	(51.1)	373.1
D9	(36.0)	(38.5)	(35.4)	(43.4)	373.6
D10	(35.8)	(38.5)	(35.5)	(43.1)	373.6
D11	48.7	(50.8)	(35.3)	(50.1)	373.1
D12	(71.7)	(74.6)	(55.6)	(64.1)	373.3
D13			72.4	(81.4)	373.1
D14	(117.0)	(113.1)	(91.2)	(99.7)	375.5
D15	(132.4)	(114.7)	(103.0)	(109.0)	373.2
D16	(167.2)	(124.8)	(122.1)	(120.8)	373.4
D17		130.3	(131.4)	(116.1)	342.0
D18				120.4	275.0

It can be seen from table 2 that since the interior of the continuous beam- arch composite bridge is high-order statically indeterminate, when a group of suspenders are tensioned, they will have a certain impact on suspenders which have been tensioned. For the proposed tensioning sequence, other suspenders have the greatest influence on the cable force before and after tensioning at D5 and D14 suspenders; the change of cable force is about 30%. The changes of suspender force of other suspenders are gentle. After the second stage pavement, the cable force reaches the reasonable completion cable force, and the accuracy meets the requirements.

In the process of suspender tension and phase II pavement, the stresses of upper and lower edges of the arch rib under various construction conditions are shown in Figure 7 and Figure 8.

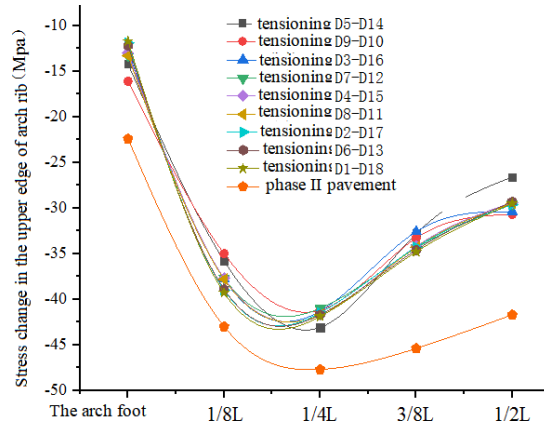


Fig.7 Stress Diagram of Upper Edge of Arch Rib.

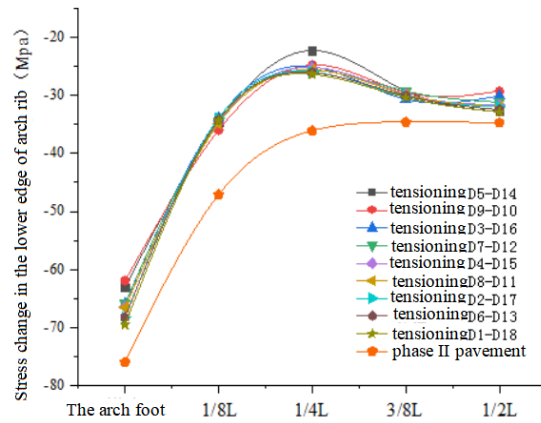


Fig.8 Stress Diagram of Lower Edge of Arch Rib.

It can be seen from Figure 7 and Figure 8 that the stresses on the upper and lower edges of the arch rib change significantly during the whole suspender tensioning process and the second phase of constant load pavement. Due to the action of the suspender tensioning, the stresses on the upper and lower edges of the arch rib at the control section will “fluctuate”. After the second phase of constant-load pavement installation, due to the increase of the suspender force, the stresses on the upper and lower edges of the arch rib in the control section increase to different degrees. The maximum variation of the stresses on the upper and lower edges of the arch rib in the control section is  $3/8L$  and  $1/4L$  respectively, and its variation values are 10.6Mpa and 9.7Mpa respectively. In the whole process of suspender tensioning and the second phase of constant load paving, the upper and lower edge stresses at the control section are within the allowable stress range of steel.

In the process of suspender tension and phase II pavement, the stresses of upper and lower edges of arch ribs under various construction conditions are shown in Figure 7 and Figure 8.

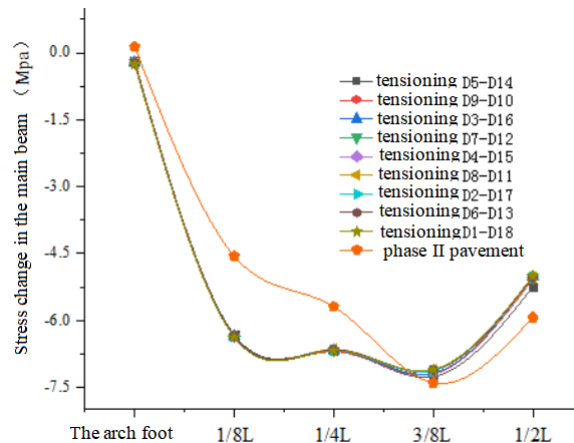


Fig.9 Stress Diagram of Upper Edge of Main Beam



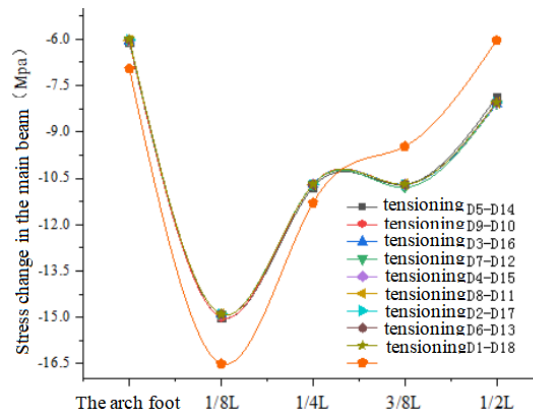


Fig.10 Stress Diagram of Lower Edge of Main Beam

It can be seen from Figure 9 and Figure 10 that the change of stresses of the upper and lower edges of the main beam are not obvious during the whole process of suspender tension, and the stress variation range of the upper and lower edges of the main beam at the control section is within 0.2MPa. After the second stage of dead load pavement, the bridge deck load changes. The stresses of the upper and lower edges of the control section beam increases or decreases in varying degrees. The maximum stress changes of the upper and lower edges of the arch rib are at the control section of  $1/8L$ , and the variation values are 1.6MPa and 1.5MPa respectively. In the whole process of suspender tension and second stage of dead load pavement, the stresses of upper and lower edges of the control section are within the allowable stress range of concrete.

## 6. Conclusion

This paper studies a new (90 + 180 + 90) m continuous beam-arch composite bridge. Combined with the structural characteristics of strong beams and weak arches and the construction method of arches after beam construction, this paper analyzes the cable force of the completed bridge by the minimum bending strain energy method. The construction cable force is analyzed by the upright method of hull section construction. The paper also analyzes the mechanical characteristics of the beam and arch structure during the suspender tension process and under the second stage dead load. The method of minimum bending energy can obtain relatively optimized and reasonable suspender cable force and completed bridge state, which has engineering value for determining the suspender cable force for railway continuous beam-arch bridges in the construction and monitoring process.

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